CHAPTER NINE

GRAVITATION, CIRCULAR MOTION AND SIMPLE HARMONIC MOTION

The angular velocity (w): This is the rate of change of angular displacement.

The period (T): This is the time interval taken by a body to complete one revolution.

N/B: V = rw, where V = linear velocity and w = angular velocity.

(Q1) A body makes six complete revolutions in 4.0 seconds. If it moves in a circle of radius 25cm, calculate

(a) the angular velocity.

(b) the velocity.

Soln:

(a)

θ = the number of revolutions = 6.

t = time in seconds = 4 seconds.

w = $\frac{\theta}{t} = \frac{6}{4} = 1.5.$

(b) Radius = r = 25cm = 0.25m.

But V = wr = 1.5×0.25

= $0.4ms^{-1}$

N/B: The angular velocity, w = $\frac{2\pi}{T}$.

Linear acceleration: This is the rate of change of linear velocity.

Angular acceleration: This is the rate of change of angular velocity.

- Now it can be proved that with reference to circular motion, $T^2 \propto r^3$ where T = the period and r = the radius.
- For this to be done, we consider the motion of a planet moving in a circle round the sun.
- If the mass of this planet is m, then the force acting on the planet = mrw^2, where r = the radius of the circle (the circular motion) and w = the angular velocity of the motion.
- But since $w = \frac{2\pi}{T}$, then the force acting on the planet = $mrw^2 = mr(\frac{2\pi}{T})^2$ $= mr(\frac{2^2\pi^2}{T^2}) = \frac{4mr\pi^2}{T^2}$
- But this force is equal to the force of attraction of the sun on the planet.
- Assuming the inverse square law holds, then the force acting on the planet $= \frac{km}{r^2}$, where k is a constant, we can say that $\frac{km}{r^2} = \frac{4\pi^2 mr}{T^2}$

  $\Rightarrow T^2 \times km = 4\pi^2 mr \times r^2$

  $\Rightarrow T^2\, km = 4\pi^2 mr^3$

  $\Rightarrow T^2 = \frac{4\pi^2 mr^3}{km} \Rightarrow T^2 = \frac{4\pi^2 r^3}{k}$

- But since π and k are constant, then it is clear that $T^2 \propto r^3$.

(Q2) What is the angular displacement in radians, when an object moves through 5m in a circular path of radius 2.5m?

Soln:

Distance = s = 5m.

Radius = r = 2.5m.

Angular displacement = $\theta = \frac{S}{r} = \frac{5}{2.5} = 2\ radians.$

(Q3) The rim of a wheel starts from rest, and after 20 seconds attains an angular velocity of 6.4 $revs^{-1}$.Determine the angular acceleration.

Soln:

Let W_0 = the original velocity and W_1 = the final velocity.

Since the body starts from rest => $w_0 = 0$. Also $w_1 = 6.4revs^{-1}$. Time = t = 20s.

Angular acceleration = a = $\frac{w_1 - w_0}{t} = \frac{6.4-0}{20}$
=> a = 2 $rads^{-2}$

N/B:

- From v = rw => w = $\frac{v}{r}$.
- The centripetal acceleration = $\frac{v^2}{r}$
- The centripetal force = $\frac{mv^2}{r}$
- Also the centripetal force = mrw^2.

(Q4) The linear velocity of a body moving round a circle of radius 5.0m is $6.5ms^{-2}$. Find

(i) its angular velocity.

(ii)the centripetal force.

Soln:

(I) V = $6.5ms^{-1}$, r = 5.0m.

w = $\frac{v}{r}$ = $\frac{6.5}{5} = 1.3\ rads^{-1}$.

(II)The centripetal acceleration = $\frac{v^2}{r}$ = $\frac{6.5^2}{5}$ = 8.5 $rads^{-2}$

(Q5) A body of mass 63kg moves round a circle of radius 9m. If the body moves with a velocity of $18kmh^1$, calculate the centripetal force required to keep the body in its circular motion.

N/B:

- Since the radius is given in metres, the velocity which is given in kilometres per hour, must be changed into metres per second.
- Since 1000m = 1km and 1 hour = $60 \times 60 = 3600\ seconds$,

=> 18km/h = $\frac{18 \times 1000}{3600}$
= $5ms^{-1}$.

Soln:

Mass = m = 63kg

r = 9m, v = $5ms^{-1}$.

If F_c = centripetal force, then $F_C = \frac{mv^2}{r} = \frac{63 \times 5^2}{9} = 175N$.

(Q6) A body of mass 5kg moves round a circle of radius 6.0m with a speed of $10.0ms^{-1}$. Calculate

(i) the angular velocity.

(ii) the centripetal force.

Soln:

(i) v = $10.0ms^{-1}$, r = 6.0m.

The angular velocity $= \frac{v}{r}$

$= \frac{10.0}{6} = 1.67 rad\ s^{-1}$.

(ii)The centripetal force $= \frac{mv^2}{r} = \frac{5 \times 10^2}{6} = 83.3N$.

(Q7) A toy car of mass 2.0kg is made to move in a circular track of radius 10.0m. If the centripetal force acting is 800.0N, determine the angular velocity of the car.

Soln:

If F_c = the centripetal force, then $F_c = mrw^2 \Rightarrow w = \sqrt{\frac{F_c}{mr}}$

Since $F_c = 800N$, m = 2kg and r = 10m, $\Rightarrow w = \sqrt{\frac{800}{2\times10}} = 20\ rads^{-1}$

(Q8) A car of mass 80kg moves in a circular track of radius 100m. If its velocity is $20ms^{-1}$, find the centripetal force which acts on the car.

Soln:

m = 80kg, v = $20ms^{-1}$, r = 100m.

If F_c = The centripetal force, then $F_C = \frac{mv^2}{r} = \frac{80\times20^2}{100}$

$\Rightarrow F_C = 320N$.

(Q9) An object of mass 50kg, moves at $5ms^{-1}$ round a circular path of radius 10m. Calculate

(a) the centripetal acceleration.

(b) the centripetal force needed to keep it in circular motion.

Soln:

(a) Centripetal acceleration $= \frac{v^2}{r} = \frac{5^2}{10} = \frac{25}{10} = 2.5ms^{-2}$.

(b)the centripetal force $= \frac{mv^2}{r} = \frac{50\times5^2}{10} = 125N$.

(Q10) A stone tied to a string is made to revolve in a horizontal circle of radius 4m, with an angular speed of 2 radians per second. With what tangential speed(velocity) will the stone move off the circle, if the string is cut.

Soln:

Tangential velocity = linear velocity, $\Rightarrow v = wr = 2 \times 4 = 8ms^{-1}$

(Q11) Find the force which is necessary to keep a mass of 0.8kg, revolving in a horizontal circular motion of radius 0.7m, with a period of 0.5 seconds.[Take $\pi^2 = 10m/s^2$].

Soln:

The needed force = F = $\frac{4mr\pi^2}{T^2} = \frac{4\times0.8\times0.7\times10}{0.5^2} = 90N$.

(Q12) What accounts for the centripetal force in the following situations?

(i) the motion of the moon around the earth.

Soln.

It is the gravitational force of attraction which constitutes the centripetal force, when the moon moves in a circular orbit around the sun.

(ii) the motion of a bus around a round about.

Soln

The frictional force acting between the wheels and the road provides the centripetal force.

(iii) the motion of a car on a banked road.

Soln

The centripetal force is due to the horizontal reactions between the surfaces in contact.

(Q13) The motion of a body along a circular path or a circle of radius 2m is defined by the equation w = $\theta^2 - 2\theta$, where w rads^{-1}and θ rad are the angular velocity and displacement respectively. The instantaneous acceleration of the body for an angular displacement θ is 24ms^{-2}. Show that θ= 3 rad.

Soln:

The angular velocity W = $\theta^2 - 2\theta$.

The radius (r) of the circle = 2m.

Instantaneous acceleration

(a) = 24ms^{-1}

w = $\sqrt{\frac{a}{r}} = \sqrt{\frac{24}{2}} = 3.46$

=> w = 3.46 rad /s.

Since W = θ^2 - 2 θ => 3.46 = θ^2 - 2 θ,=>θ^2 - 2 θ - 3.46 = 0, which is a quadratic inθ.

Solving using the formula x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

=> $\theta = \frac{-2 \pm \sqrt{2^2-4(3.46)}}{2(1)}$

=> $\theta = \frac{-2 \pm 4.22}{2}$ => $\theta = 3.11$

or θ = -1.11

Since $\theta = -1.11$ *is inadmissible*, *then* $\theta = 3.11 = 3$ rad.

N/B: The angle of inclination, or the angle that a rider must be inclined to the vertical or the angle of inclination of a track to the horizontal, to avoid sliding = θ, *where* $\tan\theta = \frac{V^2}{rg}$

(Q14) A bicycle rider traveling at a speed of 5ms^{-1}, negotiates a curve of radius 5m. Calculate the angle at which both the rider and the bicycle must incline to the vertical so as to avoid sliding. [Take g = 10ms^{-2}.]

Soln:

r = 5m, V = 5ms^{-1}, g = 10ms^{-2}

From tan $\theta = \frac{V^2}{r.g}$ => $\tan\theta$

$= \frac{5^2}{5 \text{ x } 10}$ => $\tan\theta = 0.5$,

=> θ = tan^{-1} 0.5, => θ = 26.6

N/B:

- If the reaction at the wheel = N, then Nsin θ = the centripetal force => Nsin $\theta = \frac{mV^2}{r}$
- The total reaction on the vehicle = $\frac{mg}{c0s\theta}$